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College of Eningeering
University of California, Berkeley
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STUDIES IN STABILITY

(C. A. Desoer)

(a) Refinements of previous stability results.

The general stability criterion of Desoer-Wu (1968) required that when $\hat{G}(s)$ had a pole at $s = 0$, the residue matrix R had all its eigenvalues in the open right half plane. We have been able to refine the technique so that we show that stability can be achieved once R is non-singular. We are starting to consider the case where R is singular.

Recently, in collaboration with M. Vidyasagar, we have obtained extremely general necessary conditions for the stability of a n-input n-output system where

$$e = u - G * e \qquad (u, e : \mathbb{R}_{+} \to \mathbb{R}^{n}).$$

A letter to the Editor for the Proc. IEEE is being written. A report on the other results is being prepared.

(b) Basic study for the modelling problem.

We believe that to deeply understand nonlinear dynamical systems (and especially to be able to understand, detect and circumvent quirks of such systems, as in the case of "bad" models) it is necessary to abandon the idea that the motion takes place in a linear vector space and to consider the motion as a flow on a differentiable manfold. For this purpose we are engaged in a study of differentiable geometry.

(C. A. Desoer and K. Inan)

(c) The study of the optimization of nonlinear characteristics has been completed and the results will be reported in a technical memorandum. K. Inan has obtained his Ph.D. and has left the team.

PENALTY FUNCTION METHODS

(K. Jeyarasasingam and E. Polak)

One of the major difficulties with penalty function methods is that the unconstrained optimization problem becomes progressively more and more ill conditioned as the penalty is increased. Some work to alleviate this difficulty has been done by J. Zangwill and by M. J. D. Powell. While their results are of considerable theoretical interest, the utilization of these results is far from simple, to the extent that their practical value is doubtful at present. We have been working on ways of modifying some of the ideas due to Zangwill and to Powell, in such a way as to obtain well behaved, well conditioned penalty function algorithms. This is a very difficult area and our work is proceeding rather slowly.

ALGORITHM IMPLEMENTATION

(G. Meyer, R. Klessig and E. Polak)

This work is concerned with the construction of techniques for developing stable approximations to theoretical methods. By stable approximations, we mean implementable algorithms which retain the convergence properties of the theoretical algorithms from which they are derived.

In the last six months we have prepared three reports on this subject [1]-[3]. These have also been submitted for publication. The first of these presents a new technique for constructing stable approximations. This is the last project in which G. Meyer participated. He has obtained his Ph.D. and left since. The second report uses some of our preceeding results to construct an implementable, superlinearly converging conjugate gradient method, and the third report presents a new feasible directions algorithms which uses function and gradient approximations and which can be used for solving engineering design proglems with criteria such as "minimize the rise time + 10 times the peak overshoot" of a step response, as well as constrained min max problems. At present, we are preparing a report on a gradient method for solving continuous optimal control problems, which uses an adaptive integration step size. The integration is coarse when far from the optimum and it is progressively refined as the computation proceeds. Sample tests using this algorithm show that it can be 5 - 100 times faster than the nonadaptive algorithm (currently used by everybody) from which it is evolved.

[1] R. Klessig and E. Polak, "Efficient implementation of the Polak-Ribière conjugate gradient algorithm," submitted to SIAM Journal on Control, (also Memorandum No. ERL-M279, August 1970).

[2] R. Klessig and E. Polak, "A method of feasible directions using function approximations, with applications to min max problems," Memorandum No. ERL-M287, November 1970.

[3] G. Meyer and E. Polak, "Abstract model s for the synthesis of optimization algorithms," submitted to SIAM Journal on Control, (also Memorandum No. ERL-M268, October 1969).

RATE OF CONVERGENCE OF OPTIMIZATION ALGORITHMS

(O. Pironneau and E. Polak)

A few test problems which we have constructed show that some of the best known methods of centers and of feasible directions do not converge linearly. Because of this, we have developed new modifications of these methods and have shown that, while the new methods are somewhat more laborious than the old ones, they do converge linearly. We are in the process of preparing two reports which describe this work.

The above indicated algorithms are usually used for solving non-linear programming problems. We are in the process of extending them so that they can also be used to solve certain constrained optimal control problems, and so that their linear rate of convergence is preserved.

STUDIES IN STABILITY

(D. Paranjpe and P. Varaiya)

In conventional system theory (linear and nonlinear), the state-space is always assumed to be a vector space. However, in many interesting physical systems the state is restricted to lie on a manifold in this vector space. Typical cases arise as a result of conservation laws in, say, lossless electrical networks or conservative mechanical systems. Thus, the state may be restricted to lie on a lie subgroup of the group of $n \times n$ nonsingular matrices; this group lies in the vector space of all $n \times n$ matrices. Another example is when the state is restricted to lie on an n-sphere which is diffeomorphic to the coset space $SO(n+1)/SO(n)$, where SO(n) is the set of all n × n orthogonal matrices with determinant = 1.

Hence it is interesting to study dynamical systems defined on group manifolds. R. W. Brockett has studied dynamical systems of the form

$$\dot{X} = (A_0 + \sum u_i(t)\ A_i)X + X(B_0 + \sum v_i(t)B_i)$$

where X lies in a Lie subgroup of GL(n).

We are investigating possible generalization of his results. We hope to obtain results analogous to those obtained in conventional system theory in this more general set-up.

A GENERALIZATION OF WARSHALL'S ALGORITHM

(D. Chan and L. A. Zadeh)

The well-known Warshall's algorithm [1] provides an efficient method for the computation of sums of the form

(1) $$A^* = A + A^2 + \ldots + A^d$$

where A is a square matrix of zeros and ones, d is the order of A, and the operation of addition and multiplication are Boolean. Sums of this form occur in a variety of applications, among them the analysis of flowcharts and programs, and the computation of the transitive closure of a binary relation.

The algorithm may be stated as follows. Let T_i be a transformation which takes a square matrix B into a square matrix C, with the elements of C related to those of B by the equation ($\wedge$ = Boolean product, $\vee$ = Boolean sum)

(2) $$c_{mn} = b_{mn} \vee (b_{mi} \wedge b_{in})$$

Then it can be proved that A^* is given by

(3) $$A^* = T_d \, T_{d-1} \, \ldots \, T_1 \, A$$

A problem which arises in decision-making in a fuzzy environment [2] involves the solution of an equation of the form

(4) $$w = Aw + b$$

where A is a square matrix and w and b are column-vectors whose elements are real numbers in the interval [0,1], with the operations of addition and multiplication representing $\vee$ (max) and $\wedge$ (min), respectively. It is easy to show that the solution of (4) is given by

(5) $$w = (A^{d-1} + A^{d-2} + \ldots + I)b$$

whose d is the order of A. Thus, to compute w one can employ a generalized form of Warshall's algorithm in which the elements of A are real numbers in the interval [0,1] and the operations of additions and multiplications represent $\vee$ and $\wedge$. In this algorithm, (2) is replaced by

$$c_{mn} = \text{strongest chain of length} \leq 2 \text{ passing through } i \text{ and linking } m \text{ and } n.$$

The validity of this algorithm has been established, and various applications of it are under study.

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